Marginal cost, revenue and profit

To calculate the revenue generated, the cost incurred, and the profit gained by producing x units of a product, we use the following formulas:

$R(x) = xp$ where p is the demand function for the product

$C(x) = F + V(x)$ where F is fixed cost and $V(x)$ is the variable cost

$P(x) = R(x) - C(x)$

In other words, if a company is making 100 units of their product, the revenue function will tell them how much revenue will be generated by the 100 units, the cost function will tell them how much it'll cost to produce the 100 units, and the profit function will subtract the total cost from the total revenue to given them the profit gained from producing and then selling the 100 units.

But what if the company thinks it can make more money if it produces and sells more than 100 units. Well, depending on the cost associated with producing more units, it may or may not be profitable to produce and sell more units than they already are.

To decide whether or not to ramp up production, the company can use *marginal* revenue, *marginal* cost, and *marginal* profit functions, which are the derivatives of the original three functions. When the company finds the marginal functions and evaluates them at 100 units, the current level of production, the marginal functions will tell them the additional revenue generated, the additional cost incurred, and the additional profit gained from producing and selling the 101st unit.

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This should make sense to us, because all we're saying is that the derivatives (the marginal functions) tell us the *change* per unit, and we know that derivatives in general tell us the rate of change of the function.

Example

A smart phone manufacturer knows that the cost of producing x phones is given by

$$C(x) = 6x^2 + 34x + 2{,}500$$

and that the demand function for their phones is

$$p = 60x$$

Calculate the marginal cost, marginal revenue and the marginal profit of producing 75 phones.

To calculate **marginal cost** at 75 units, we take the derivative of the cost function and then evaluate the derivative at $x = 75$.

$$C'(x) = 12x + 34$$

$$C'(x) = 12(75) + 34$$

$$C'(x) = 934$$

The marginal cost at $x = 75$ is $\$934$, which means the additional cost associated with producing the 76th unit is $\$934$.

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To calculate **marginal revenue** at 75 units, we need to find a revenue function, take its derivative, and then evaluate the derivative at $x = 75$.

The revenue equation is $R(x) = xp$ where p is the demand function, $p = 60x$.

$$R(x) = x(60x)$$

$$R(x) = 60x^2$$

Taking the derivative of revenue to get marginal revenue, we get

$$R'(x) = 120x$$

Then we evaluate at $x = 75$.

$$R'(x) = 120(75)$$

$$R'(x) = 9{,}000$$

The marginal revenue at $x = 75$ is $\$9{,}000$, which means the additional revenue associated with selling the 76th unit is $\$9{,}000$.

Finally, to solve for **marginal profit** we need to find a profit function, take its derivative, and then evaluate the derivative at $x = 75$.

The profit equation is $P(x) = R(x) - C(x)$, where R is the revenue function we found earlier and C is the cost function we were given.

$$P(x) = \left(60x^2\right) - \left(6x^2 + 34x + 2{,}500\right)$$

$$P(x) = 54x^2 - 34x - 2{,}500$$

Taking the derivative of profit to get marginal profit, we get

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$$P'(x) = 108x - 34$$

Then we evaluate at $x = 75$.

$$P'(x) = 108(75) - 34$$

$$P'(x) = 8{,}066$$

The marginal profit at $x = 75$ is $\$8{,}066$, which means the additional profit associated with producing and selling the 76th unit is $\$8{,}066$.

Remember that marginal cost, revenue and profit are always only approximations. For that reason, $P'(x)$ will not always be equal to $R'(x) - C'(x)$.

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